Unsteady mixed convection boundary layer stagnation point flow and heat transfer in a nanofluid over a stretching sheet

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ABSTRACT-This paper deals with unsteady mixed convection boundary layer stagnation point flow and heat transfer of nanofluid over a stretching sheet. The physical aspects of the problem are pointed and discussed. The influencing partial differential equations are detracted to non-linear differential equation using symmetry transformation. Circumstantially; the key focus is on heat transfer on copper, Titanium and silver water nanofluids. Most erudite problems are projected by non-linear ordinary or partial differentiation. The numerical outcomes are examined for various parameters, such as Prandtl number, mixed convection parameter, the solid volume fraction of nanoparticles and unsteadiness parameter.

Keyword: boundary layer, stretching sheet,mixed convection, nanofluid, stagnation point, bvp4c

1 INTRODUCTION

A comprehensive study has been accomplished on boundary layer flow over a stretching sheet since the pioneering work done by Sakiadis [1]. The problem of unsteady mixed convection boundary layer flow and heat transfer has both logical and practical values. Convective heat transfer in nanofluid is a topic of considerable concern in science and engineering. Modern heat transfer industries depend upon high performance heat transfer accessories. Maxwell [2] was the first to give concept of improving heat and mass transfer performance of fluid with the inclusion of solid particles. Most scientific problems are designed by non-linear ordinary of partial differential. Boundary layer flows can be takenas an example. The study on the different method used for solving the non-linear differential equations is an important topic for the analysis of engineering practical problem. Common fluids such as water, oil, glycol and ethylene have poor heat transfer characteristics. Because of their low thermal conductivity of these fluids is to suspended nanosized metallic particles such as copper, gold, iron, aluminum, titanium or their oxides in the fluid to enhance its thermal properties can provide fair improvements in the thermal conductivity and in the heat transfer coefficient of the base fluids. Nanofluids are actually homogenous mixture of base fluid and nanoparticles.Nanoparticles range in diameter between 1 and 100nm. Nanofluids commonly contain up to a 5% volume fraction of nanoparticles to ensure effective heat transfer enhancements. Nanofluid concept was first coined by Choi [3]He showed that the addition of a small amount (less than 1% by volume) of nanoparticles to conventional heat transfer liquids enhanced the thermal conductivity of the fluid up to approximately two times. The written work on nanofluid has been reconsidered by Trisakri and Wongwise [4]. These reviews examined the work done on convective transport in nanofluids. Nanofluids are described by enhanced thermal conductivity; a phenomenon observed by Masuda et al [5].The makes nanofluids attractive for multifarious engineering applications involving cooling and heat exchange including advanced nuclear system [6]. Heat transfer characteristics of nanofluids were reviewed by Wang and Majumdar [7]. Numerical studies on natural convection heat transfer in nanofluids have been discussed in [8-16]. Tiwari and Das [17] analyzed the behavior of nanofluids taking into account the solid volume fraction. The comprehensive references on nanofluids can be found in the recent book by Das et al [18].Nadeem and Hussain[19] have solved analytically the problem of MHD flow of a viscous fluid on a non-linear porous shrinking sheet. The study on shrinking sheet was first initiated by Wang [20] by considering the stretching declaration surface for various controlling parameters and magnetic parameters. The aim of the present paper is to study the consequences of various parameters on unsteady mixed convection boundary layer flow and heat transfer of nanofluid over a stretching sheet in existence of Copper Cu, Silver Ag and Titanium oxide TiO_2 nanoparticles.

2BASIC EQUATIONS

Consider the unsteady two- dimensional mixed convectional boundary layer flow due to a stretching/shrinking sheet in a incompressible fluid. The fluid is a water based nanofluids containing three types of nanoparticles such as Copper Cu , Silver Ag and Titanium oxide TiO_2. It is also assumed that the base fluid and nanoparticles are in thermal equilibrium and no slip occurs between them. The thermo physicalproperties of regular fluid and nanoparticles are given in Table 1.

Thermo-physical properties of water and nanoparticles.

Physical properties	Regular fluid(water H2O)	Copper(Cu)	Silver(Ag)	Titanium (TiO2)
cp(J/kg K)	4179	385.0	235	686.2
ρ (kg/m3)	997.1	8933	10500	4250
k (W/m K)	0.613	400	429	8.9538
β $\times$10-5(1/K)	21	1.67	1.89	0.90

The governing equations of the present problems are:

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \qquad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U_\infty\frac{dU_\infty}{dx}+\frac{dU_\infty}{dt}+\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2}+g\frac{\phi\rho\beta_s+(1-\phi)\rho\beta_f}{\rho_{nf}}(T-T_\infty) \qquad (2)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2} \qquad (3)$$

Where u and v are velocity components along the axes x and y respectively, g is acceleration due to gravity, β_f and β_s are the thermal expansion coefficients of the base fluid and the nano-solid particles respectively. ϕ is the solid volume fractions of nanoparticles, μ_{nf} the dynamic viscosity of the nanofluid, T the temperature of fluid and ρ_{nf} is the effective density of the nanofluid.

The corresponding boundary conditions for velocity and temperature are

$$u=U_w(x),\quad v=0;\quad T=T_w \quad at\ \ y=0$$
$$u\to U(x),\quad T\to T_\infty \quad as\ \ y\to\infty \qquad (4)$$

Following Ishak et al [21] the stretching velocity is considered as $U_w(x,t)=\frac{ax}{1-ct}$ where a and c are constant (with $a\geq 0$, $c\geq 0$ such that $ct\prec 1$) and both have dimension t^{-1}, we have a as the initial stretching rate $\frac{a}{1-ct}$ and it increases with time. We considered the surface temperature $T_w(x,t)$ the stretching sheet to vary with the distance x and an inverse law for its decrease with time as:

$$T_w=T_\infty+\frac{bx}{(1-ct)^2}.$$

Now introducing the following similarity transformations

$$\eta=\sqrt{\frac{a}{\nu}}(1-ct)^{-\frac{1}{2}}y,$$
$$\psi=(a\nu)^{\frac{1}{2}}(1-ct)^{\frac{-1}{2}}xf(\eta)$$
$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \qquad (5)$$

here ψ is the stream function that satisfies Equation (1) with

$$u=\frac{\partial\psi}{\partial y} \quad \text{and} \quad v=-\frac{\partial\psi}{\partial x} \qquad .(6)$$

In terms of these variables the velocity components can be expressed as,

$$u=\frac{ax}{1-ct}f'(\eta), \qquad v=-\sqrt{\frac{a\nu_f}{1-ct}}f(\eta) \qquad (7)$$

Now for nanofluids, let us introduce the expressions for ρ_{nf}, $(\rho C_p)_{nf}$, α_{nf}, μ_{nf} of the nanofluid as

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s,\quad \mu_{nf}=\mu_f(1-\phi)^{-2.5},\quad \alpha_{nf}=\frac{k_{nf}}{(\rho Cp)_{nf}}$$
$$(\rho Cp)_{nf}=(1-\phi)(\rho Cp)_f+\phi(\rho Cp)$$

(8)

The thermal conductivity of the nanofluid k_{nf} can be calculated by the formula

$$\frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)} \quad (9)$$

The momentum and energy equations together with the boundary conditions given by (4) can be written

$$f''' - (1-\phi)^{2.5}\left[\begin{array}{l}\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)\left\{A\left(f'+\frac{\eta}{2}f''-1\right)+f'^2-ff''-1\right\} \\ -\xi\theta\left(1-\phi+\phi\frac{\rho\beta_s}{\rho\beta_f}\right)\end{array}\right] = 0 \quad (10)$$

$$\theta'' - \Pr\frac{k_f}{k_{nf}}\left\{1-\phi+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right\}\left\{f'\theta - f\theta' + A(2\theta+\tfrac{1}{2}\eta\theta')\right\} = 0 \quad (11)$$

where $\Pr = \frac{\nu}{\alpha}$ the Prandtl is number, $A = \frac{c}{a}$ is dimensionless measure of the unsteadiness and $\xi = \frac{g\beta_f b}{a^2}$ is the buoyancy or mixed convection parameter.

The transformed boundary conditionsare:

$$f'(0) = \lambda,\; f(0) = 0,\; \theta = 1 \text{ at } \eta = 0 \quad (12)$$

$$f'(0) = 1,\; \theta = 0 \text{ at } \eta \to \infty,$$

where $\lambda = \frac{b}{a}$ is the stretching/shrinking parameter according as $\lambda \rangle 0$ or $\lambda \langle 0$.

The physical quantities of interest are the skin friction coefficient and the local Nusselt number, which are defined as

$$C_f = \frac{\mu}{\rho f U_w^2}\left(\frac{\partial u}{\partial y}\right)_{y=0};$$

$$Nu = \frac{x}{k(T_w - T_\infty)}\left[k\left(\frac{\partial T}{\partial y}\right)_{y=0} - \frac{4\sigma}{3k_2}\left(\frac{\partial T^4}{\partial y}\right)_{y=0}\right] \quad (13)$$

with μ and k are the dynamic viscosity and thermal conductivity, respectively. Using non-dimensional variables (7), we have

$$C_f \operatorname{Re}_x^{1/2} = F''(0), \quad \frac{Nu_x}{\operatorname{Re}_x^{1/2}} = -\theta''(0) \quad (14)$$

3 RESULTS AND DISCUSSION

The boundary value problem (10)-(12) has been solved numerically with the MATLAB function "bvp4c". The function has three input variables: the name of the M-file enumerating an ordinary differential equation system of the design, the term of the M-file enumerating the boundary values, and an initial approximation of the result prepared with the MATLAB function "bvpinit". The output variable of "bvp4c" answers the solution in the form $[f(\eta), f'(\eta), f''(\eta), f'''(\eta)]$, which can be calculated at any given framework with the MATLAB function "deval". This paper discussed about the effect of solid volume fraction ϕ, Prandtlnumber $\Pr$, buoyancy or mixed convection ξ and unsteadiness A on unsteady mixed convection boundary layer flow of nanofluid over a stretching/shrinking sheet.Table-2shows computational values $-f''(0)$ and $-\theta'(0)$ for Copper, Titanium and Silver nanoparticles with different values ϕ.

Table 2: Values of $-f''(0)$ and $-\theta'(0)$ for different values of ϕ when A=0.1, ξ =0.2, Pr=1

ϕ	$-f''(0)$			$-\theta'(0)$		
	Copper (Cu)	Titanium (TiO_2)	Silver(Ag)	Copper (Cu)	Titanium (TiO_2)	Silver(Ag)
0.0	-1.297050	-1.297050	-1.297050	0.888321	0.888321	0.888321
0.1	-1.545031	-1.286738	-1.568662	0.825331	0.795293	0.808989
0.2	-1.543026	-1.219736	-1.674686	0.720659	0.707184	0.730053
0.3	-1.521258	-1.169669	-1.584285	0.652573	0.646479	0.622087
0.4	-1.387823	-1.034093	-1.445077	0.565908	0.570337	0.534310

For the validation of the study, the case when the combined magnetic and porosity parameter is absent has been also considered and compared with the results reported by Ramachandran et al.[22] , Lok et al[23] and Ishak [24]. This comparison is shown in Table 3.

Table 3: comparison table of $-f''(0)$ for various values of Pr

	Pr	Ramachandran et. al. [22]	Lok et. al.[23]	Ishak et. al.[24]	Present study
$\xi=1$	0.7	1.7063	1.706376	1.7063	1.706303
	7	1.5179	1.517952	1.5179	1.517913
	40	1.4101	1.410094	1.4104	1.410068
	60	1.3903	1.390311	1.3903	1.390278
	100	1.3680	1.368070	1.3680	1.368043

Effect of pertinent parameters on the velocity profile –

Effect of nanoparticles solid volume fraction parameter ϕ:
The velocity profile for different varying parameter is depicted in fig 1-3. Fig 1 is for variation in solid volume fraction ϕ. In fig-1 when $A=2$, and $\xi=1$. The velocity profile increases to a certain level and it is maximum for $\phi=0.3$ Silver nanofluid and again it decreases to 1.

Effect of unsteadiness parameter A:
The variation in unsteadiness parameter A represented in Fig 2. From the figures it is apparent that the velocity profile increases to a certain level and again decreases asymptotically to 1. It is observed that increasing unsteadiness parameter results in a decrease in the thermal boundary layer, thickness, associated with an increase in the wall temperature gradient

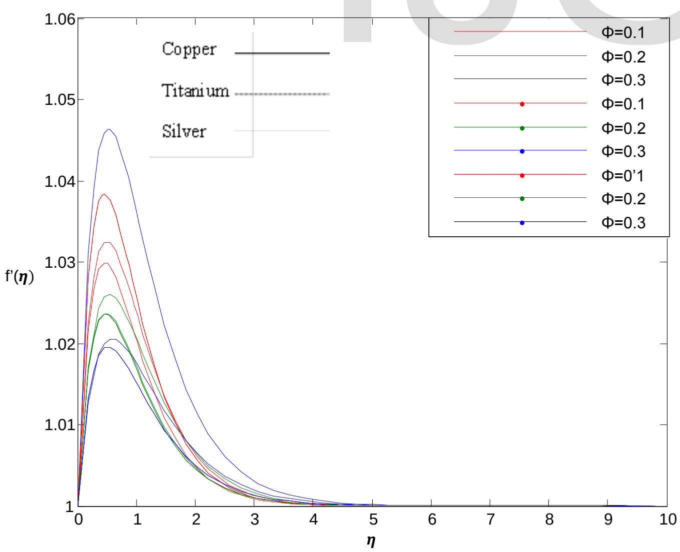
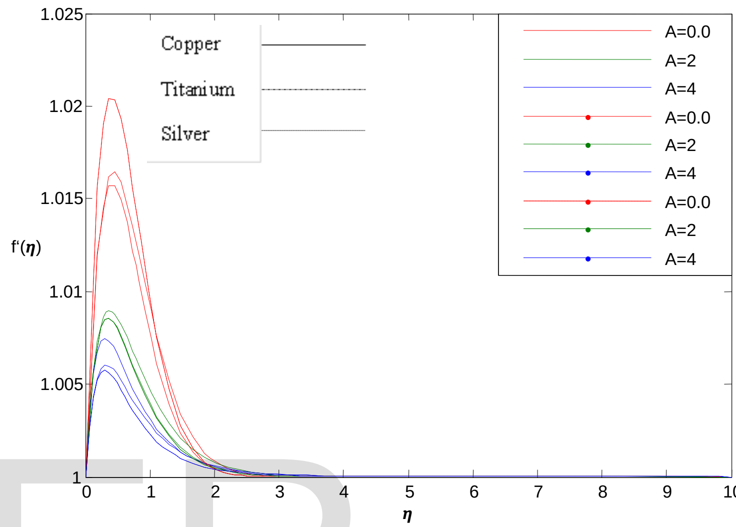


Figure 1: ϕ=0.1, 0.2, 0.3, □=1, A=2, Pr=1, Velocity profile for different values of solid volume fraction parameter ϕ



Figurer2: A=0, 2, 4, □=0.5, ϕ=0.1, Pr=3, Velocity profile for different values of unsteadiness parameter A

Effect of mixed convection parameter ξ:

Fig3 is for mixed convection parameter ξ in the presence of the nanoparticles Cu, Ag and TiO_2 Fig3 depicted that the velocity profile increases to a certain level and again it decreases asymptotically to 1. Fig shows that when $\phi=0.1$, $A=4$ the velocity profile increases to a level and again decreased to 1.

Effect of pertinent parameters on the temperature profile –

Effect of nanoparticles solid volume fraction parameter ϕ:

Figure4 shows the effect of volume fraction ϕ on temperature profile with different values of unsteadiness parameter A. From the figure it is obvious that the temperature profile increases with the increase in solid volume fraction ϕ. The thermal boundary layer increases with the increase in volume fraction of the nanoparticles.

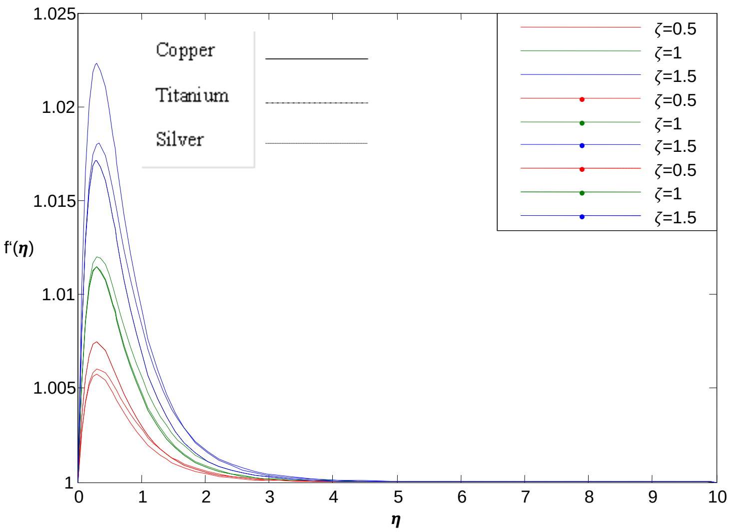


Figure3: □=0.5, 1, 1.5, A=4, Pr=3, ϕ=0.1 Velocity profile for mixed convection parameter

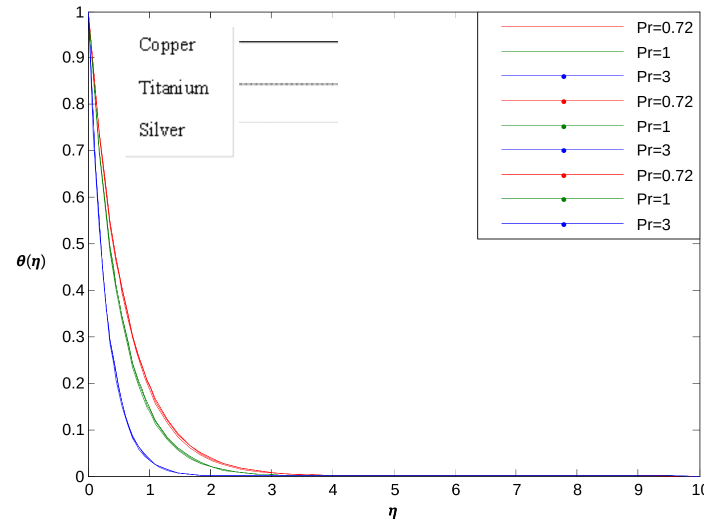


Figure5: Pr=0.72, 1, 3, A=2, ϕ=0.1, Temperature profile for different values of Prandtl number

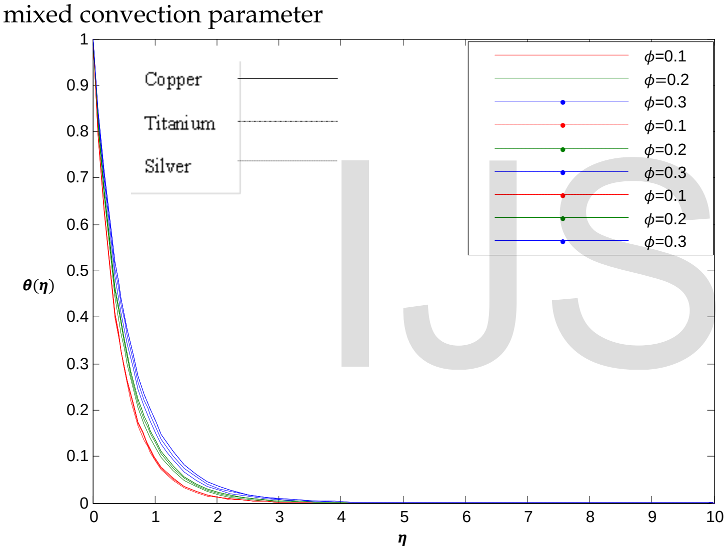
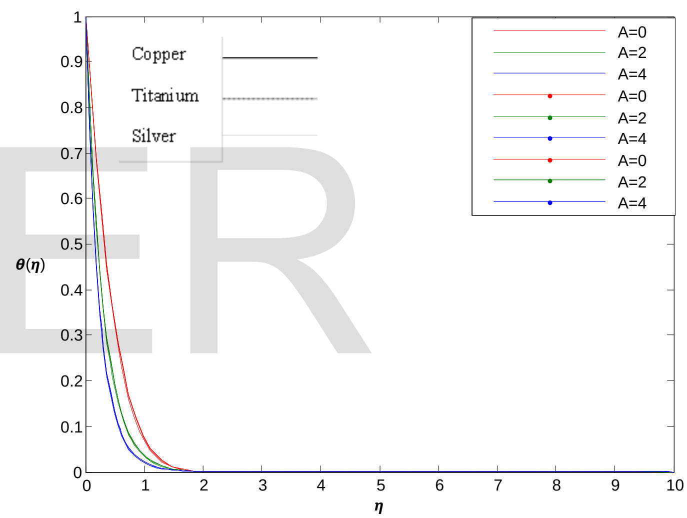


Figure 4:ϕ=0.1,0.2,0.3, □=1, A=4, Pr=3, Temperature profile for solid volume fraction parameter



Figurer6: A=0, 2, 4, ϕ=0.1, Pr=3, Temperature profile for different values of unsteadiness parameter A

Effect of PrandtlnumberPr:

Figure 5 exhibits the effect of variation in Prandtl number on temperature profile with different values of A, the unsteadiness parameter. From the figure it is obvious that the temperature profile decreases with the increase in Prandtl number. The thermal boundary layer thickness decreases with the increase in Prandtl number.

Effect of unsteadiness parameter A:

Figure6 is are for variation in unsteadiness parameter A on temperature profile. From these figure it is clear that the increase in unsteadiness parameter A, there is a decrease in thermal boundary layer thickness.

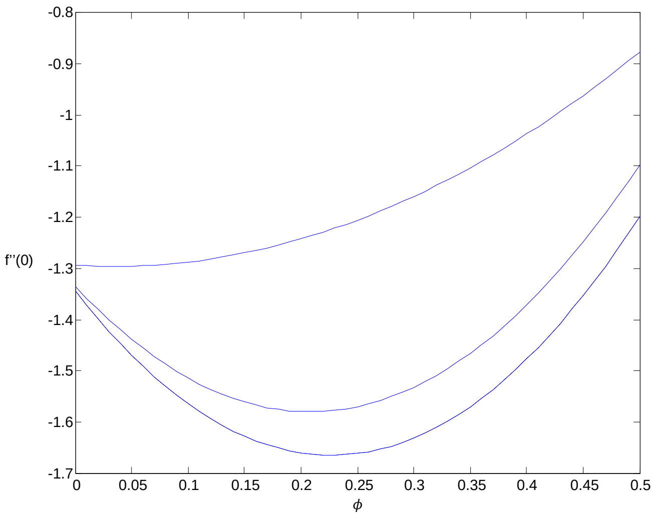


Figure 7: A=0.1, ζ=0.2, Pr=1, Skin friction against different values of ϕ

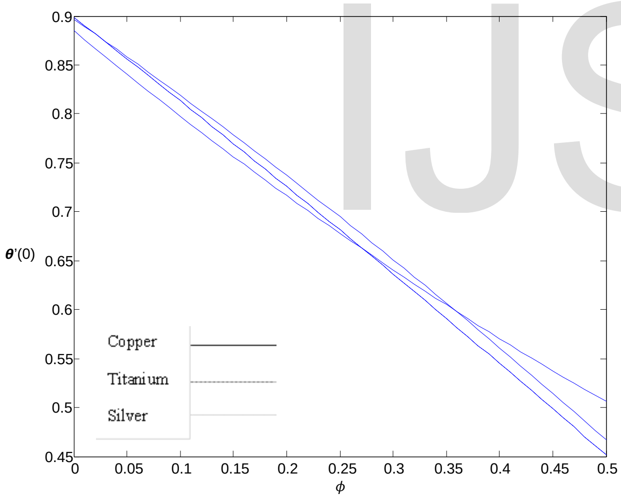


Figure 8: A=0.1, ζ=0.2, Pr=1, Nusselt number against different values of ϕ

4 CONCLUSION

The unsteady two dimensional mixed convection boundary layer flow by cause of a stretching surface take advantage of nanofluid has been reviewed. We discussed the effects of the governing parameter A, ϕ, Pr, ξ and different nanofluids Cu, Ag, TiO_2:

- The thermal boundary layer increases with the increase in volume fraction of the nanoparticles.
- The thermal boundary layer thickness decreases with the increase in Prandtl number.
- The present result has been compared with Ramachandran[22],Lok [23] and Nazar [24]. The uniformity between the results is excellent.

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